Grand Prize

Duality

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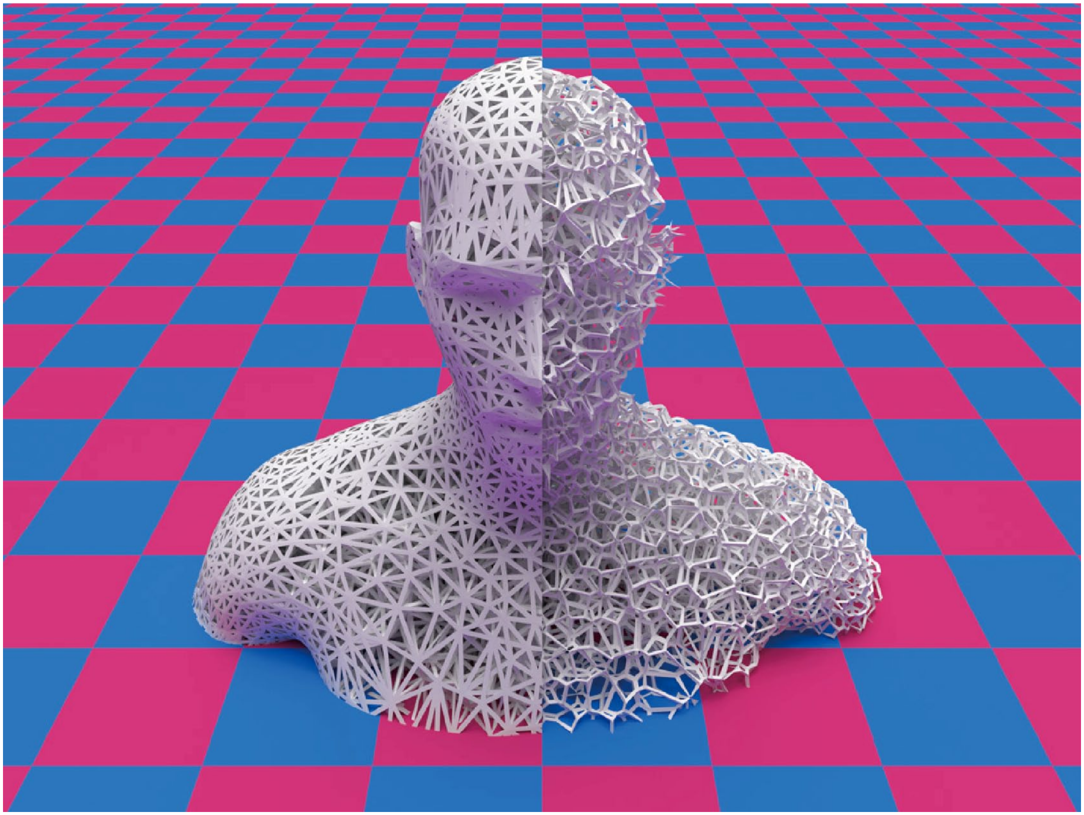
Masanori Nakayama, Issei Fujishiro "Duality" 2017, Material: VeroWhite, Size: 246W x 120D x 189H (mm)

In the present work, we attempted at expressing human "two-facedness" by the "two-sidedness" of the 3D structures. In the field of geometry, there exists a two-facedness called "duality," which represents the relationship between diagrams that switch between faces (cells) and vertices. Delaunay diagram and Voronoi diagram give a typical example. We have developed an algorithm to generate a Poisson-disk distribution in an arbitrary shape to make sculptures with those diagrams. The Delaunay diagram generated by Poisson-disk distribution point cloud makes up the uniform tetrahedral mesh (except for the surface). Since those cells are close to a regular tetrahedron, Voronoi vertices located on the circumcenters also have an almost uniform distribution.

Keywords: Tetrahedralization, Delaunay diagram, Voronoi diagram



1 Concept

We have worked on an optimal subdivision algorithm to discretize a given arbitrary shape. In 2D, we built on Delaunay diagram to subdivide an arbitrary closed curve region into a uniform triangular mesh with Poisson disk distribution (Fig. 1). By embedding the 2D Delaunay diagram in 3D, we succeeded to subdivide an arbitrary closed surface volume into a tetrahedral mesh with Poisson disk

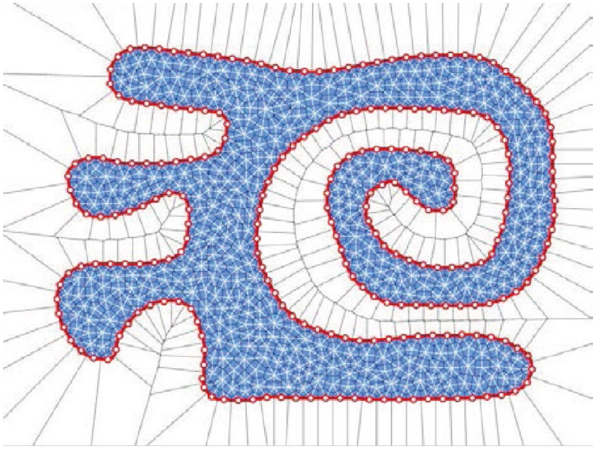


Figure 1: Poisson-disk triangulation.

distribution (Fig. 2). In comparison with Guo's prior work [2], the incorporated non-polygonal isosurfacing increases the versatility in terms of input volume, simplifies meshing steps, and improves the quality of output meshes.

All of our works root commonly in the "duality" of Delaunay diagram and Voronoi diagram—both structures are mutually convertible by swapping the roles of faces and vertices. In the present work, we attempted at expressing human "two-facedness" with the "two-sidedness" of the two diagrams.

2 How to Make

There exists a de facto standard tetrahedralization library called *Tetgen* [3]. For edgy curve surfaces that can be defined in a piecewise manner, a specific type of tetrahedralization algorithm was devised in [4].

In our tetrahedralization scheme, we first generate a Delaunay diagram with vertices on the target's surface, and thereby making separate inside and outside of the target

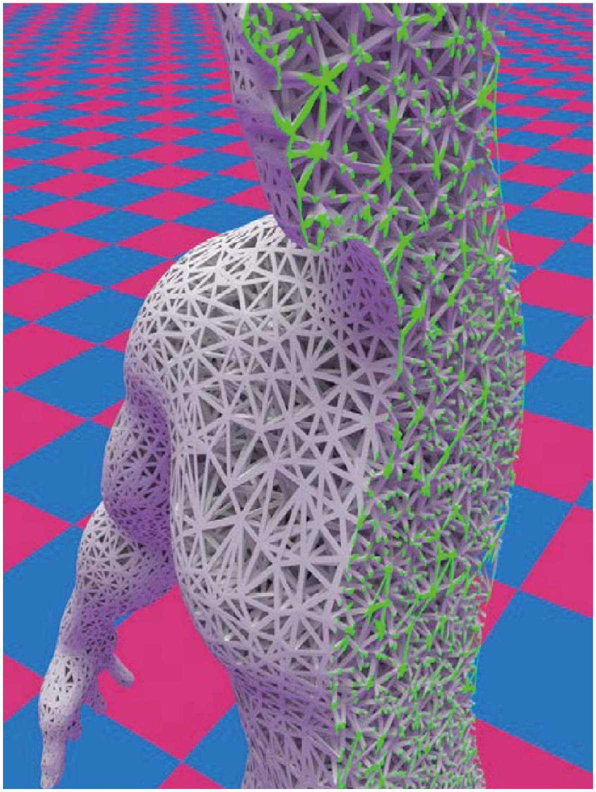


Figure 2: Poisson-disk tetrahedralization.

shape. Fig. 3 simply represents this in 2D.

Next, we add points only in the inside region. Note that in order to uniformly distribute vertices, we should not insert them randomly. For each tetrahedron, we search for the largest circumscribed sphere, and insert a point at the center of the sphere (Fig. 4). The position of the point is nothing less than the vertex of the Voronoi diagram, indicating the most distant location from the existing points in the largest blank area.

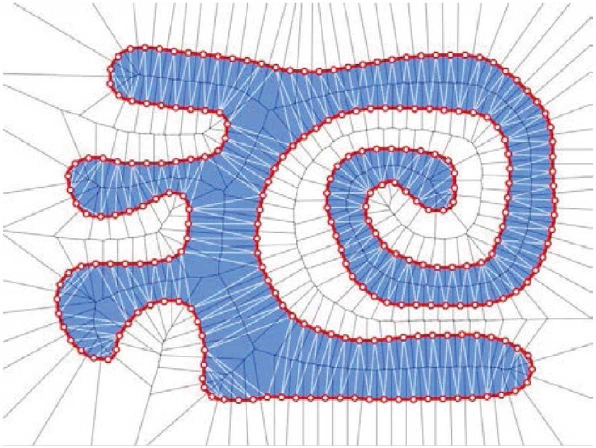


Figure 3: Delaunay diagram of edge points.

By repeating the process, the vertex distribution automatically converges to a Poisson disk distribution, and resulting in the most uniform tetrahedral mesh. Variational tetrahedral meshing in [1] can further improve the location of all the vertices gradually, but works slowly.

In the process, the Voronoi diagram is generated together with the Delaunay diagram, whereas the Voronoi diagram itself has no utility value. In the present work, however, by juxtaposing the two diagrams, we intended to express the interest of geometry.

3 Software & System

All the programs have been implemented in the *Object Pascal* language using an integrated development environment *Embarcadero Delphi*. For editing/rendering the model, the 3DCG software *Cinema 4D* was used.

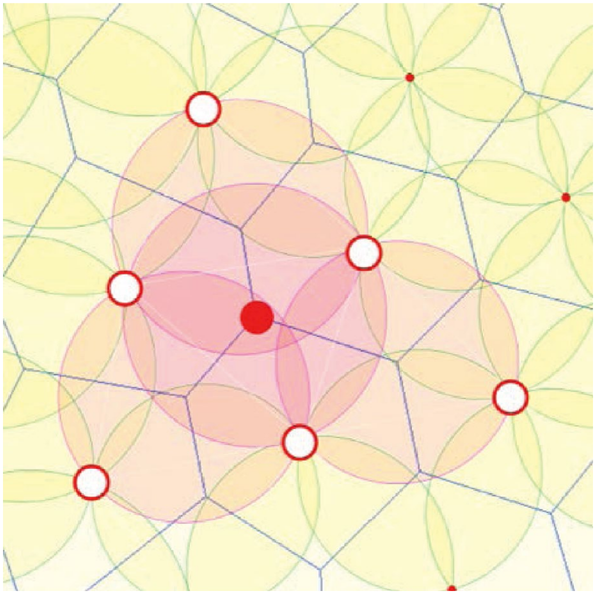


Figure 4: Voronoi vertices in empty circle.

References

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